SUPERSONIC FLOW PAST TRIANGULAR WINGS WITH RIBS ON THEIR SURFACE

PMM Vol. 31, No. 6, 1967, pp. 1050-1056

A.L. GONOR
(Moscow)

(*Received July 20, 1967*)

Various types of partitions are a common feature of lifting surfaces. These partitions can take the form of stiffening ribs, deflectors for preventing secondary flows or flow separation, etc. The presence of partitions has a marked effect on the character of flow and on the values of the aerodynamic parameters. Flow past such wings cannot be computed in the general case. Wings of a special type are amenable to simple solution, however, and this will be considered below. One special case of interaction between a partition and an infinite wing is also considered in [1].

1. Formulation of the problem and construction of the solution. Let us consider a triangular conical wing with supersonic leading edges. We assume that the portion of the wing adjoining the edges is flat. We shall seek our solution in the coordinate system r, θ, φ shown in Fig. 1. The relationship between our coordinates and the Cartesian ones is given be Expressions [2]

$$x = r\cos\theta\,\sin\varphi, \qquad y = r\sin\theta, \qquad z = r\cos\varphi\,\cos\theta$$

The conical flows in terms of these variables are described by the system

$$v\frac{\partial u}{\partial\theta} + \frac{w}{\cos\theta}\frac{\partial u}{\partial\varphi} - v^2 - w^2 = 0, \qquad v\frac{\partial v}{\partial\theta} + \frac{w}{\cos\theta}\frac{\partial v}{\partial\varphi} + uv + w^2\operatorname{tg}\theta = -\frac{1}{\rho}\frac{\partial p}{\partial\theta}$$

$$\frac{\gamma}{\gamma-1}\frac{p}{\rho} + \frac{u^2+v^2+w^2}{2} = \frac{1}{2} + \frac{1}{(\gamma-1)M_\infty^2} = \frac{c}{2} \tag{1.1}$$

$$v\frac{\partial}{\partial\theta}\frac{p}{\rho^\gamma} + \frac{w}{\cos\theta}\frac{\partial}{\partial\varphi}\frac{p}{\rho^\gamma} = 0$$

$$2\rho u + v\frac{\partial\rho}{\partial\theta} + \frac{w}{\cos\theta}\frac{\partial\rho}{\partial\varphi} + \frac{\rho}{\cos\theta}\left[\frac{\partial(v\cos\theta)}{\partial\theta} + \frac{\partial w}{\partial\varphi}\right] = 0$$

The first two of these equations constitute projections of the Euler equations on the r and θ axes. The three latter equations are the energy, entropy, and particle mass conservation equations. All of the variables in the equations are dimensionless, and u, v, and w, i.e. the velocity components along the r, θ, and φ axes, respectively, refer to the velocity U at infinity; ρ is the density referred to the density ρ^0 of the free stream; p is the pressure referred to the velocity head $\rho^0 v^2$; γ is the adiabatic exponent; M_∞ is the Mach number of the unperturbed stream. We shall solve the problem for the following boundary conditions.

1°. The relations

$$U_n = U_n^*\rho^*, \qquad U_{\tau_1} = U_{\tau_1}^*, \qquad U_{\tau_2} = U_{\tau_2}^*, \qquad \varepsilon = \frac{\gamma-1}{\gamma+1}$$

$$\rho^* = \frac{1}{\varepsilon}\left[1 + \frac{2}{(\gamma-1)M_\infty^2 U_n^2}\right], \qquad p^* = \frac{2}{\gamma+1}U_n^2 - \frac{\varepsilon}{\gamma M_\infty^2} \tag{1.2}$$

must be fulfilled at the shock wave whose surface equation we denote by $\theta^*(\varphi)$.

The quantities with the subscripts n, τ_1, and τ_2 are the projections of the velocity on the normal and on the two mutually perpendicular tangents to the surface of the shock wave.

2°. On the body surface defined by Eq. $\theta_1(\varphi)$ we have $U_n = 0$. The boundary conditions on the flat portion of the wing near the edges are $v = 0$, $\theta = 0$. Now let us make use of (1.2) to express the values of all the required functions at the shock wave in terms of the equation of the wave surface. To this end we introduce the three unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ along the axes r, θ, and φ, respectively. In the Cartesian coordinate system shown in Fig. 1 these are defined by the following projections: (1.3)

$$\mathbf{e}_1\,[\cos\theta\sin\varphi,\ \sin\theta,\ \cos\theta\cos\varphi],\ \mathbf{e}_2\,[-\sin\theta\sin\varphi,\ \cos\theta,\ -\sin\theta\cos\varphi],\ \mathbf{e}_3\,[\cos\varphi,\ 0,\ -\sin\varphi]$$

This new basis enables us to determine readily the projections of the unit vectors tangent and normal to the surface of the shock wave.

In fact, in the left-handed system we have

$$\boldsymbol{\tau}_1 = \mathbf{e}_1(1,0,0), \qquad \boldsymbol{\tau}_1 \times \mathbf{n} = \boldsymbol{\tau}_2\,[0,\ -q(1+q^2)^{-1/2},\ -(1+q^2)^{-1/2}]$$

$$\mathbf{n}\,[0,\ (1+q^2)^{-1/2},\ -q(1+q^2)^{-1/2}], \qquad q = \theta_\varphi^* / \cos\theta^* \tag{1.4}$$

Relations (1.2) to (1.4) enable us to represent the values of the required functions at the shock wave as

$$u^* = \cos\alpha\cos\theta^*\cos\varphi - \sin\alpha\sin\theta^*, \qquad v^* = \frac{Aq+B}{1+q^2}, \qquad w^* = \frac{A-Bq}{1+q^2}$$

$$U_n = \frac{-1}{\sqrt{1+q^2}}\,[(\sin\alpha\cos\theta^* + \cos\alpha\sin\theta^*\cos\varphi) - q\cos\alpha\sin\varphi] \tag{1.5}$$

$$A = -q(\cos\theta^*\sin\alpha + \sin\theta^*\cos\alpha\cos\varphi) - \cos\alpha\sin\varphi$$

$$B = -\varepsilon\left[1 + \frac{2}{(\gamma-1)M_\infty^2U_n^2}\right](\sin\alpha\cos\theta^* + \cos\alpha\sin\theta^*\cos\varphi - q\cos\alpha\sin\varphi)$$

We begin to construct the solution with a study of flow past the flat portion of the wing in the neighborhood of the leading edge. Fig. 2 is a diagram of this flow with a plane shock OAC attached to the wing edge OAB. The velocities of the free stream and of the

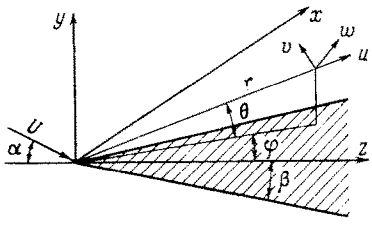

Fig. 1

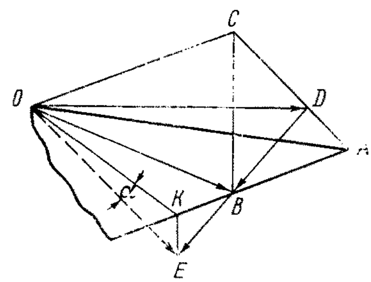

Fig. 2

homogeneous flow behind the shock are represented by the segments OE and OB; the velocity component tangent to the shock is represented by the segment OD. We can readily see that all of the planes passing through the line OB and intersecting the shock are stream surfaces. Hence, the plane OBC normal to the wing can be considered as an impermeable barrier (at which $w = 0$). Further, we can assume that the flow in the outer zone between the wing and barrier is determined by the uniform flow behind the plane shock. Although the flow behind the plane shock has been thoroughly investigated, its application to flow past a conical wing enables us to make several remarks of theoretical interest. Let us consider the velocity triangle OED in Fig. 2. This triangle includes the normal component DB of the velocity behind the shock, which is equivalent to the projection of the velocity normal to the radius OD. Recalling that DB is always smaller than the velocity of sound, we obtain the following inequality at the point D:

$$v^2 + w^2 < a^2 \tag{1.6}$$

Here a is the velocity of sound in the stream behind the shock. Let us trace the change in the velocity normal to the radius accompanying passage from the point D to the point B. According to Fig. 2 it decreases monotonously to zero, so that inequality (1.6) remains valid at all points of the segment BD. On the other hand, by the condition of the problem the edge OA is supersonic; the velocity component normal to the radius is larger than the velocity of sound at the point A. This yields an inequality opposite to (1.6) in the neighborhood of the point A. Let us write out the equations of the acoustic characteristics of system (1.1). We have

$$\frac{d\theta}{d\varphi} = \frac{-vw \pm a\sqrt{v^2 + w^2 - a^2}}{a^2 - w^2}$$

The change of sign of inequality (1.6) alters system of differential Eqs. (1.1) (e.g. see [3]). Thus, the translational stream on the outer portion of the wing in conical variables constitutes transonic flow and can be considered as a particular analytic solution of a mixed system. In this solution the velocity components u, v, w (p and ρ are constants) and the sonic line can be determined directly from Expressions

$$u = U_1 \cos\theta \cos(\varphi - \varphi_1)$$

$$v = -U_1 \sin\theta \sin(\varphi - \varphi_1), \qquad w = -U_1 \sin(\varphi - \varphi_1)$$

$$\cos\theta = \frac{(1 - M_1^{-2})^{1/2}}{\cos(\varphi - \varphi_1)}$$

where U_1 and M_1 are the velocity and the Mach number of the homogeneous stream behind the shock; φ_1 is the coordinate of the point B (Fig. 2). For certain values of the geometric parameters and Mach numbers a second parabolic line in the neighborhood of the point C arises in addition to the parabolic line in the neighborhood of the point A. The fact is that the Mach cone for a uniform stream behind the shock can intersect the flow zone in two places. It is interesting to note that there are no singularities on the sonic line in this case.

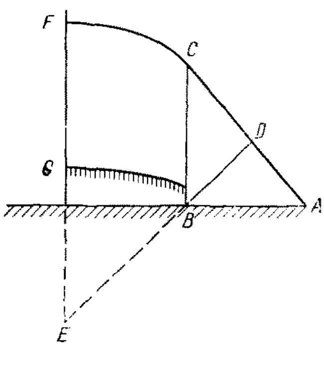

Fig. 3

Let us now attempt to find the solution in the inner zone between the partition BC and the plane of symmetry GF (Fig. 3). On the lines BC and GF we have the condition $w = 0$, at the shock wave FC conditions (1.2) and (1.5), and at the wall GH the condition of streamline flow. Let us assume that the solution of the problem can be obtained in the class of flows for which $w \sim \delta \sim 0$ in the entire domain $CFGH$. In this case the unknown functions u, v, p and ρ (in accordance with (1.1)) satisfy the following system of equations:

$$\frac{\partial u}{\partial\theta} - v = 0, \tag{1.7}$$

$$v\frac{\partial v}{\partial\theta} + uv = -\frac{1}{\rho}\frac{\partial p}{\partial\theta}\quad \frac{\gamma}{\gamma - 1}\frac{p}{\rho} + \frac{u^2 + v^2}{2} = \frac{c}{2}$$

$$\frac{p}{\rho^\gamma} = \delta^\gamma(\varphi), \qquad 2u + v\frac{\partial \ln\rho}{\partial\theta} + \frac{1}{\cos\theta}\frac{\partial(v\cos\theta)}{\partial\theta} = 0$$

The second Eq. of (1.1) has been omitted, since it is a consequence of Eqs. (1.7); $\delta(\varphi)$ is an arbitrary function. The pressure is given by Formula

$$p = \left[\frac{1}{2}(c - u^2 - u_0^2)\frac{\gamma - 1}{\gamma\delta}\right]^{\gamma/(\gamma-1)}, \qquad \delta(\varphi) = \frac{p^{*1/\gamma}}{\rho^*} \tag{1.8}$$

The velocity u can be determined on the basis of system (1.7) from the quasilinear second-order equation

$$u_{\theta\theta}(1-M_n^2)-u_\theta \operatorname{tg}\theta+2u(1-{}^1/_2 M_n^2)=0$$

$$M_n^2=\frac{2u_\theta^2}{(\gamma-1)(c-u^2-u_\theta^2)}=\frac{v^2}{a^2} \tag{1.9}$$

The coefficient M_n is the Mach number in the transverse flow on the sphere $r=$ const. We can show that $M_n^2<1$ for sufficiently large M_∞. In fact,

$$M_n^2 \sim v^{*2}/a^{*2} \sim \varepsilon(k_1+|q|k_2), \qquad 0<k_i<1$$

If $|q|$ is on the order of unity, then $M_n<1$; if $|q|$ is large, then $M_n>1$.

Let us consider the boundary conditions in more detail. The equation $w^*=0$ must be fulfilled at the shock wave. By virtue of (1.5) this means that $A-Bq=0$ and that the surface of the shock wave can be found by solving the first-order differential Eq.

$$a_0\theta_\varphi^{*3}+a_1\theta_\varphi^{*2}+a_2\theta_\varphi^*+a_3=0, \quad a_0=\varepsilon b^2+\varepsilon_1, \quad a_1=ab(1-2\varepsilon)\cos\theta^*$$

$$a_2=-[a^2(1-\varepsilon)-b^2-\varepsilon_1]\cos^2\theta^*$$

$$a_3=-ab\cos^3\theta^*, \qquad a=\cos\theta^*\sin\alpha+\sin\theta^*\cos\alpha\cos\varphi$$

$$b=\cos\alpha\sin\varphi, \qquad \varepsilon_1=2/(\gamma+1)M_\infty^2 \tag{1.10}$$

Equation (1.10) must be solved under the condition that the shock wave passes through the point $C(\varphi_1, \theta_1^*)$. It is interesting to note that for $M_\infty=\infty$ Eq. (1.10) breaks down into the two simpler equations

$$(b\theta_\varphi^*-a\cos\theta^*)\ (\theta_\varphi^{*2}\ \varepsilon b+\theta_\varphi a(1-\varepsilon)\ \cos\theta^*+b\cos^2\theta^*)=0 \tag{1.11}$$

The first factor cannot be a solution because the symmetry condition $\theta_\varphi^*(0)=0$ is not satisfied; the second factor yields two negative roots. The smaller of these roots, i.e. the one which corresponds to a Mach number $M_n<1$ (see (1.9)), is the one required. For the second root we have $M_n>1$, and, as we shall show, no solution exists.

If the point C is determined by the condition of flow of the translational stream past the outer portion of the wing with the attached shock AC, then adjunction of the shock AC with the curved shock CF automatically preserves the continuity of the derivative. Thus, the shock wave can be determined unambiguously from Eq. (1.10) or (1.11) and from the point C. By virtue of (1.5) and (1.7) this has the effect of stipulating that

$$u(\theta^*,\varphi)=u^*, \qquad u_\theta(\theta^*,\varphi)=v^*=B \tag{1.12}$$

at the points of the shock wave.

In relation to Eq. (1.9) these functions constitute boundary conditions which determine a unique solution. Hence, the streamline flow condition $v=u_\theta=0$ is extraneous and cannot be satisfied on an arbitrary surface $\theta_1(\varphi)$. The only remaining alternative is to construct the solution of Eq. (1.9) under conditions (1.12) and to determine whether its domain of existence includes a surface at whose points $u_\theta=0$. If it does, then the shape of the inner portion GH of the wing is determined (Fig. 3) and the solution becomes closed. The flow at the inner portion of the wing can be of independent interest in the case of ribbed-edge wings or of conical channels with flat side walls. The latter occur in the design of air intakes. The solution constructed for the class of flows with $w\equiv 0$ satisfies system (1.1) and all the boundary conditions. However, in this class of solutions system (1.1) degenerates, and the first three equations are not equivalent to three Euler equations. This means that system (1.1) is not complete in some cases.

2. Investigation of the solution and computed results. Conical flow near the wings considered above exists if the Cauchy problem with the appropriate streamline flow condition is solvable for Eqs. (1.9) to (1.11). Investigation of Eq. (1.11) enables us to establish that the domain of existence of the solution is defined by the inequality

$$(\sin\alpha\cos\theta+\cos\alpha\sin\theta\ \cos\varphi)^2-(\gamma^2-1)\ \cos^2\alpha\ \sin^2\varphi\geqslant 0 \tag{2.1}$$

If the plane shock is attached to the wing edge, then this inequality is certainly fulfilled at the point $C(\varphi_1, \theta_1^*)$ of the rib. In fact, it is easy to show that it is fulfilled over the entire range $0 \leqslant \varphi \leqslant \varphi_1$. On the other hand, if we are considering the inner flow at a wing with an arbitrary apex angle φ_1 and a rib $\theta_1{}^*$, then fulfillment of the inequality at the initial point $(\theta_1{}^*, \varphi_1)$ once again guarantees the existence of a solution for the shock wave. In this case the permissible values of the initial parameters $(\theta_1{}^*, \varphi_1, \alpha)$ are bounded by the surface defined by relation (2.1) taken with the equal sign.

Examples of such domains for several values of the angle α appear in Fig. 4. The domain of permissible parameters can be determined in a similar way in the case of finite Mach numbers.

Now let us investigate Equation (1.9).

We begin with the case $M_n < 1$, first rewriting the equation in the form

$$u_{\theta\theta} = \frac{-u\,(2 - M_n{}^2) + u_\theta \operatorname{tg} \theta}{1 - M_n{}^2} \tag{2.2}$$

Proceeding from (2.2) and recalling that the inequalities $u > 0$ and $u_\theta < 0$ are fulfilled in the flow domain, we find that $u_{\theta\theta} < 0$. Let us first consider the stream field in the domain adjacent to the rib $\varphi = \varphi_1$. We stipulate that $u^*(\theta_1{}^*, \varphi_1) > 0$ and $u_\theta{}^*(\theta_1{}^*, \varphi_1) < 0$ at the point of intersection of the rib and shock wave. By virtue of the inequality for the second derivative the curve of the radial velocity $u(\theta, \varphi_1)$ is convex and has a shape like that of the lower curve in Fig. 5. In order to determine whether the derivative u_θ vanishes for some value of θ, we shall compare it with the radial velocity distribution in the uniform stream adjacent to the right side of the rib $\varphi = \varphi_1$. By virtue of (1.8), the velocity u° of the homogeneous stream satisfies Eq. $u_{\theta\theta}{}^\circ = -u^\circ$. Comparing the latter with (2.2), we obtain the inequality $|u_{\theta\theta}| > |u_{\theta\theta}{}^\circ|$. On the other hand, at the initial point $(\theta_1{}^*, \varphi_1)$ we have $u = u^\circ$, $u_\theta = u_\theta{}^\circ$; we also know that $u_\theta{}^\circ = 0$ for $\theta = 0$. This means that the derivative u_θ vanishes for $\theta > 0$. The distribution of the velocity u° of the uniform stream appears as the broken curve in Fig. 5. Without going into detail, we note that the condition $u_\theta = 0$ is fulfilled for positive θ throughout the range $0 \leqslant \varphi \leqslant \varphi_1$. Computations show that the wing contour GH is convex, as is shown in Fig. 3.

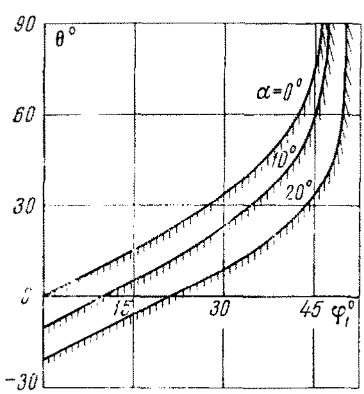

Fig. 4

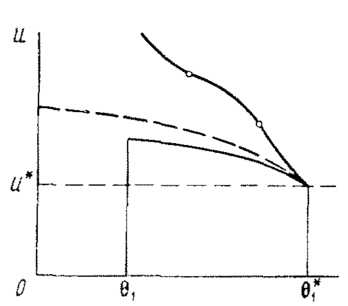

Fig. 5

Let us now turn to the case $M_n > 1$ and assume to begin with that $1 < M_n < 2$. Eq. (2.2) then yields the condition $u_{\theta\theta} > 0$, which implies an increase in the absolute value of u_θ as the argument decreases (see the upper curve in Fig. 5). Increases in the derivative $|u_\theta|$ can be accompanied by increases in the coefficient M_n such that beginning with some value of θ the quantity M_n becomes much larger than two. This makes possible a change in the sign of $u_{\theta\theta}$ in Formula (2.2) and the appearance of an inflection point in the curve $u(\theta)$ after which $|u_\theta|$ begins to diminish. It can never reach zero, however, since for $M_n < 1$ we have another inflection point and $|u_\theta|$ once again increases, etc. Hence, the resulting class of conical flows corresponds to flow past the wing only for $M_n < 1$.

Qualitative analysis of the velocity distribution at the rib enables us to draw a useful conclusion: namely, that according to Fig. 5 the quantities u and u_θ on the inner side of the rib are smaller than their respective values on the outer side. This implies that the pressure force (see (1.8)) acting on the inner portion HC of the rib (Fig. 3) is larger than that acting on the outer portion. Hence, the total forces applied to the ribs produce a thrust which reduce the total drag.

Of the greatest interest in this connection are narrow wings with ribs on their leading edges (see the first diagram in Fig. 6). With such an arrangement the pressure on the outer side of the rib is much smaller than on the inner side. The domain of higher pressure covers the entire wing, so that the thrust and lift are increased more markedly.

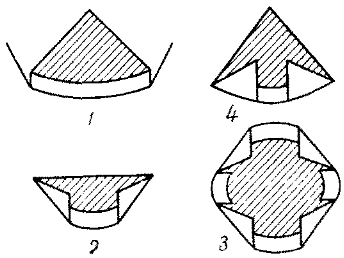

Fig. 6

Now let us consider the computed results. The equations were integrated by the Runge-Kutta method using a standard computer routine. The first step was to solve the Cauchy problem for Eq. (1.10) with the data ($\theta_1{}^*$, φ_1) chosen either arbitrarily or by computing the translational streamline flow past the outer portion of the wing. The shape $\theta^*(\varphi)$ of the shock wave and its derivative $\theta_\varphi^*(\varphi)$ were determined in the range $0 \leqslant \varphi \leqslant \varphi_1$ in the course of the computations. The next step was to use (1.12) and (1.5) to find the initial conditions for Eq. (1.9) which contained φ as the parameter. Eq. (1.9) was integrated in five planes $\varphi = \varphi_i$ which divided the interval from 0 to φ_1 into equal parts. The functions sought were $u(\theta, \varphi_i)$ and $u_\theta(\theta, \varphi_i)$. Computation was terminated upon the attainment by the second function of zero to within six places. This procedure enabled us to find five points of the wing contour. The pressure coefficient $C_p = 2p$ was computed at the wall and rib using Formula (1.8).

The results obtained in computing the contours of the body and shock wave in the transverse cross section $x = 1$ for the $M_\infty = \infty$, $\gamma = 1.4$, $\alpha = 10°$ and for two initial positions of the rib (10°, 5°) and (5°, 5°) appear in Fig. 7. It is interesting to note for large apex angles φ the body contour is highly curved, while for small apex angles it is almost straight. Fig. 8 (broken curves) shows the pressure distributions at the wall and rib. Regardless of the contour curvature, the pressure at the wall is almost constant and increases towards the plane of symmetry of the wing. At the rib the pressure increases slightly towards the point of adjunction with the body surface.

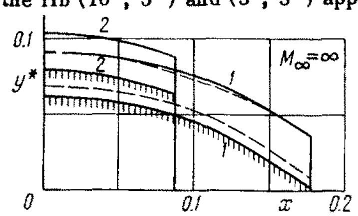

Fig. 7

The computed data for a finite Mach number, $M_\infty = 3$, $\alpha = 6°30'(8°10', 26°36')$ appear in Fig. 9. The broken curve represents the pressure distribution at the wall. The qualitative behavior of the curves described above remains the same in this case. Constant pressure at the wing means that the velocity w is, in fact, small, and that our solution is highly accurate.

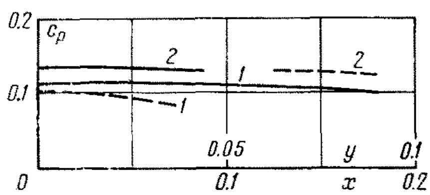

Fig. 8

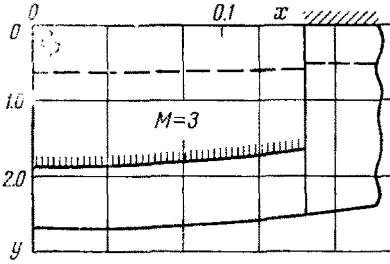

Fig. 9

The class of flows characterized by a velocity $w \sim \delta$ enables us to investigate not only past a wing with the transverse cross section shown in Fig. 3. Taken together with the translational stream, it can be used to determine the flows near other wings and bodies, e.g. near those whose cross sections appear in Fig. 6.

3. Hypersonic flow past a wing. Let the Mach number of the free stream be large ($M_\infty = \infty$) and let the density ratio determined by the parameter be $\varepsilon \ll 1$. The coordinate system r, θ, φ (Fig. 1) in this case can be conveniently related to the surface of the body by measuring the angle θ from one of the generatrices of the body, e.g. from the value $\theta = \theta_1(\varphi_1)$. Then instead of the coordinate θ we have the new angle ϑ related to the former by the expression $\theta = \vartheta + \theta_1(\varphi_1)$. If we measure the angle of attack α° from the plane $\vartheta = 0$, then we must replace α by the difference $(\alpha^\circ - \theta_1)$ in all expressions.

In the hypersonic approximation, assuming that the wave is sufficiently smooth (narrow wings), we have the estimates

$$\vartheta \sim \varepsilon, \quad u \sim 1, \quad v \sim \varepsilon, \quad p \sim 1, \quad \rho \sim \varepsilon^{-1}$$

From this, retaining only the principal terms in (1.10), we obtain the following equation for the shock wave:

$$\frac{d\vartheta^*}{d\varphi} + \frac{\operatorname{ctg}(\alpha^\circ - \theta_1)\sin\varphi}{1 + \operatorname{tg}\theta_1 \operatorname{ctg}(\alpha^\circ - \theta_1)\cos\varphi} = 0$$

The two terms in the left-hand side must be of the same order. Hence, $\varphi \sim \varepsilon^{1/2}$, and the approximation is valid only for sufficiently narrow wings.

The function $\vartheta^*(\varphi)$ is given by the relation

$$\vartheta^* = \vartheta_1^* + \operatorname{ctg}\theta_1 \ln\left[\frac{1 + \operatorname{tg}\theta_1 \operatorname{ctg}(\alpha^\circ - \theta_1)\cos\varphi}{1 + \operatorname{tg}\theta_1 \operatorname{ctg}(\alpha^\circ - \theta_1)\cos\varphi_1}\right] \qquad (3.1)$$

Eq. (1.9) and boundary conditions (1.12) can be simplified in the same way. Retaining the principal terms, we find that

$$u_{\vartheta\vartheta} + 2u = 0, \qquad u^* = \cos(\alpha^\circ - \theta_1)\cos\theta_1\cos\varphi - \sin(\alpha^\circ - \theta_1)\sin\theta$$

$$u_\vartheta^* = -\varepsilon\,[\sin(\alpha^\circ - \theta_1)\cos\theta_1 + \cos(\alpha^\circ - \theta_1)\sin\theta_1\cos\varphi] \qquad (3.2)$$

We can write the solution as

$$u = a\sin\sqrt{2}\,\vartheta + b\cos\sqrt{2}\,\vartheta, \quad a = u^*\sin\sqrt{2}\,\vartheta^* + u^*/\sqrt{2}\cos\sqrt{2}\,v^*, \quad b = u^*\cos\sqrt{2}\,v^*$$

From this and from the streamline flow condition we find that the wing contour is given by Expression

$$\vartheta_1(\varphi) = \vartheta^*(\varphi) - \frac{\varepsilon}{2}\,\frac{\sin(\alpha^\circ - \theta_1)\cos\theta_1 + \cos(\alpha^\circ - \theta_1)\sin\theta_1\cos\varphi}{\cos(\alpha^\circ - \theta_1)\cos\theta_1\cos\varphi - \sin(\alpha^\circ - \theta_1)\sin\theta_1} \qquad (3.3)$$

The parameter θ_1 appearing in the above formulas can be determined from the condition $\vartheta(\varphi_1) = 0$.

If we assume, moreover, that the wing is slender ($\theta_1 \sim \varepsilon$) and measure the angle ϑ from the plane $\theta_1 = 0$, then instead of (3.1) we obtain Expressions

$$\vartheta^* = \vartheta_1^* + \operatorname{ctg}\alpha^\circ(\cos\varphi - \cos\varphi_1), \qquad \vartheta_1 = \vartheta^* - {}^1/_2\,\varepsilon\operatorname{tg}\alpha^\circ / \cos\varphi \qquad (3.4)$$

which indicate that the wing contour is convex.

Computations carried out using Formulas (3.4) appear as broken curves in Fig. 7. Comparison indicates good agreement with the results computed for $M = \infty$.

The author is grateful to G.G. Chernyi and G.N. Andreev for their useful comments on the present paper.

BIBLIOGRAPHY

1. A.I. Golubinskii and A.N. Ivanov, Some exact solutions of the problem of flow of a supersonic and a hypersonic gas past a slipping wing. Izv. Akad. Nauk SSSR, MZhG, No. 1, 1966.
2. A.L. Gonor, High-velocity supersonic gas flows past conical bodies. Izv. Akad. Nauk SSSR, Mekhanika i Mashinostroenie, No. 1, 1959.
3. B.M. Bulakh, Some questions concerning the theory of conical flow. PMM Vol. 25, No. 2, 1961.

Translated by A.Y.